DISTRIBUCIONES BIDIMENSIONALES

(EJERCICIOS RESUELTOS DEL TEXTO)

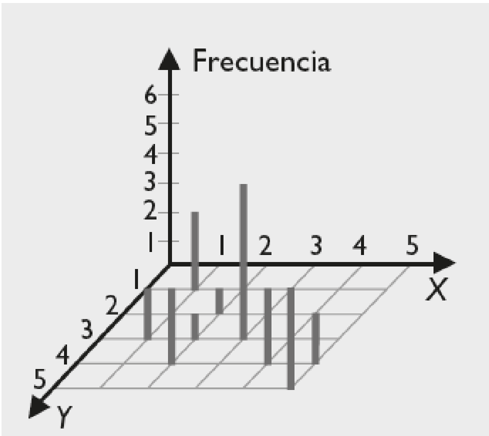
1.

1.er examen	4	5	6	7	7	9	10
2.º examen	5	5	7	6	7	8	10
N.º estudiantes	5	10	4	2	4	3	2

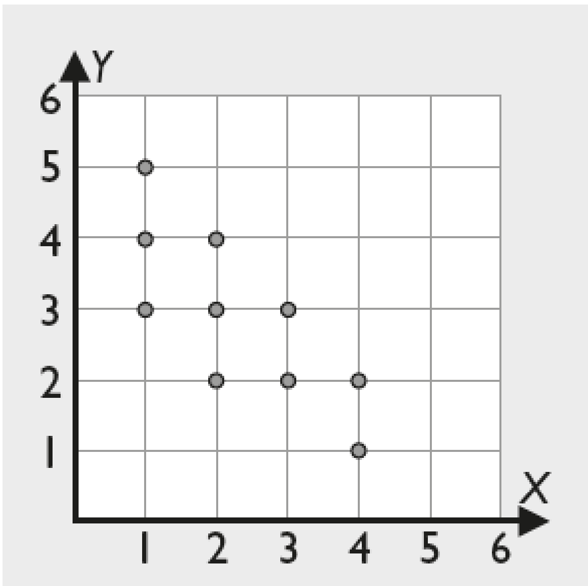
Y \ X	4	5	6	7	9	10	
5	5	10	0	0	0	0	15
6	0	0	0	2	0	0	2
7	0	0	4	4	0	0	8
8	0	0	0	0	3	0	3
10	0	0	0	0	0	2	2
	5	10	4	6	3	2	30

2.

Y \ X	1	2	3	4	5
1	3	0	0	0	0
2	0	1	0	0	0
3	2	1	6	0	0
4	0	3	0	3	2
5	0	0	0	0	4



3.



4.

x_i	8	7	6	5	7	8	6	5
y_i	5	4	7	4	3	6	5	5
n_i	2	4	3	5	3	4	2	2

X_i	Y_i	n_i	$X_i \cdot n_i$	$Y_i \cdot n_i$	$X_i \cdot Y_i \cdot n_i$
8	5	2	16	10	80
7	4	4	28	16	112
6	7	3	18	21	126
5	4	5	25	20	100
7	3	3	21	9	63
8	6	4	32	24	192
6	5	2	12	10	60
5	5	2	10	10	50
		25	162	120	783

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{162}{25}=6{,}48 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{120}{25}=4{,}8$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{783}{25}-6{,}48 \cdot 4{,}8 \approx 0{,}22$$

5.

X_i	Y_i	n_i	$X_i \cdot n_i$	$Y_i \cdot n_i$	$X_i \cdot Y_i \cdot n_i$
2	1	1	2	1	2
2	2	2	4	4	8
2	3	3	6	9	18
4	1	3	12	3	12
4	2	5	20	10	40
4	3	1	4	3	12
4	4	2	8	8	32
6	2	1	6	2	12
6	3	4	24	12	72
8	1	2	16	2	16
8	3	6	48	18	144
		30	150	72	368

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{150}{30}=5 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{72}{30}=2{,}4 \qquad \sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{368}{30}-5\cdot 2,.4\approx 0{,}27$$

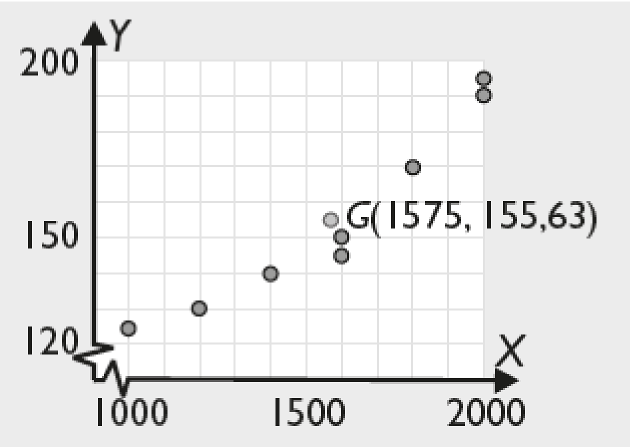
6.

X_i	Y_i	n_i	$X_i \cdot n_i$	$Y_i \cdot n_i$	$X_i \cdot Y_i \cdot n_i$
1000	125	1	1000	125	125.000
1200	130	1	1200	130	156.000
1400	140	1	1400	140	196.000
1600	145	1	1600	145	232.000
1600	150	1	1600	150	240.000
1800	170	1	1800	170	306.000
2000	190	1	2000	190	350.000
2000	195	1	2000	195	390.000
		8	12600	1245	2.025.000

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{12600}{8}=1575 \quad cm^3 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{1245}{8}=155{,}625 \quad Km/h$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{2.025.000}{8}-1575\cdot 155{,}63\approx 8015{,}63$$

Nube de puntos orientada hacia arriba y derecha.



7.

X_i	1	4	4	2	5	3	1	20
Y_i	5	2	3	6	3	2	4	25
n_i	1	1	1	1	1	1	1	7
$X_i^2 \cdot n_i$	1	16	16	4	25	9	1	72
$Y_i^2 \cdot n_i$	25	4	9	36	9	4	16	103
$X_i \cdot Y_i \cdot n_i$	5	8	12	12	15	6	4	62

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{20}{7}=2{,}86 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{25}{7}=3{,}57$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{72}{7}-2{,}86^2}=1{,}45$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{103}{7}-3{,}57^2}=1{,}40$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{62}{7}-2{,}86\cdot 3{,}57\approx -1{,}35 \qquad r=\frac{\sigma_{xy}}{\sigma_x \sigma_y}=\frac{-1{,}35}{1{,}45\cdot 1{,}4}\approx -0{,}66$$

Correlación débil e inversa.

8.

X_i (ºC)	10	12	14	15	17	20	88
Y_i (€)	150	120	102	90	50	18	530
n_i	1	1	1	1	1	1	6
$X_i^2 \cdot n_i$	100	144	196	225	289	400	1354
$Y_i^2 \cdot n_i$	22.500	14.400	10.404	8.100	2.500	324	58.228
$X_i \cdot Y_i \cdot n_i$	1.500	1.440	1.428	1.350	850	360	6.928

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{88}{6}=14{,}67\,^{o}C \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{530}{6}=88{,}33\,€$$

$$\sigma_x = \sqrt{\frac{\Sigma x_i^2 \cdot n_i}{N} - \overline{x}^2} = \sqrt{\frac{1354}{6} - 14{,}67^2} = 3{,}2538\ ^{\circ}C$$

$$\sigma_y = \sqrt{\frac{\Sigma y_i^2 \cdot n_i}{N} - \overline{y}^2} = \sqrt{\frac{58.228}{6} - 88{,}33^2} = 43{,}6174\ €$$

$$\sigma_{xy} = \frac{\Sigma x_i \cdot y_i \cdot n_i}{N} - \overline{x} \cdot \overline{y} = \frac{6928}{6} - 14{,}67 \cdot 88{,}33 \approx -141{,}1344 \qquad r = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \approx -0{,}99$$

Correlación muy fuerte e inversa

9.

Y \ X	1	2	3	4
1	1	2	0	0
2	2	1	0	0
3	0	1	2	3
4	0	4	3	1

X_i	1	1	2	2	2	2	3	3	4	4	
Y_i	1	2	1	2	3	4	3	4	3	4	
n_i	1	2	2	1	1	4	2	3	3	1	20
$X_i \cdot n_i$	1	2	4	2	2	8	6	9	12	4	50
$Y_i \cdot n_i$	1	4	2	2	3	16	6	12	9	4	59
$X_i^2 \cdot n_i$	1	2	8	4	4	16	18	27	48	16	144
$Y_i^2 \cdot n_i$	1	8	2	4	9	64	18	48	27	16	197
$X_i \cdot Y_i \cdot n_i$	1	4	4	4	6	32	18	36	36	16	157

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{50}{20}=2{,}5 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{59}{20}=2{,}95$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{144}{20}-2{,}5^2}=0{,}9747$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{197}{20}-2{,}95^2}=1{,}0712$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{157}{20}-2{,}5\cdot 2{,}95\approx 0{,}475 \qquad r=\frac{\sigma_{xy}}{\sigma_x \sigma_y}\approx 0{,}4546$$

10.

X_i	1	2	3	4	5	15
Y_i	26	30	27	31	28	142
n_i	1	1	1	1	1	5
$X_i^2 \cdot n_i$	1	4	9	16	25	55
$Y_i^2 \cdot n_i$	676	900	729	961	784	4050
$X_i \cdot Y_i \cdot n_i$	26	60	81	124	140	431

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{15}{5}=3 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{142}{5}=28{,}4$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{55}{5}-3^2}=1{,}4142$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{4050}{5}-28{,}4^2}=1{,}8547$$

$$\sigma_{xy}=\frac{\Sigma x_i\cdot y_i\cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{431}{5}-3\cdot 28{,}4\approx 1 \qquad r=\frac{\sigma_{xy}}{\sigma_x\sigma_y}=\frac{1}{1{,}4142\cdot 1{,}8547}\approx 0{,}3812$$

Correlación débil y directa.

Rectas de regresión: $$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-2{,}84=\frac{1}{2}(x-3)\rightarrow y=0{,}5x+26{,}9$$

y=0,5x+26,9 (y sobre x)

$$x-\bar{x}=\frac{\sigma_{xy}}{\sigma_y^2}\cdot(y-\bar{y})\rightarrow x-3=0{,}291(y-28{,}4)\rightarrow x=0{,}291y-5{,}264$$

x=0,291y-5,264 (x sobre y)

11.

X_i (mg)	100	200	300	400	500	600	2100
Y_i (hrs)	4	3,5	3	2	2,5	1,5	16,5
n_i	1	1	1	1	1	1	6
$X_i^2\cdot n_i$	10.000	40.000	90.000	160.000	250.000	360.000	910.000
$Y_i^2\cdot n_i$	16	12,25	9	4	6,25	2,25	49,75
$X_i\cdot Y_i\cdot n_i$	400	700	900	800	1250	900	4950

$$\bar{x}=\frac{\Sigma x_i\cdot n_i}{N}=\frac{2100}{6}=350\ mg \qquad \bar{y}=\frac{\Sigma y_i\cdot n_i}{N}=\frac{16{,}5}{6}=2{,}75\ hrs$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{910.000}{6}-350^2}=170{,}7825\ mg$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{49{,}75}{6}-2{,}75^2}=0{,}8539\ hrs$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{4950}{6}-350\cdot 2{,}75\approx -137{,}5 \qquad r=\frac{\sigma_{xy}}{\sigma_x\sigma_y}\approx -0{,}9429$$

Correlación muy fuerte e inversa

Recta de regresión:

$$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-2{,}75=\frac{-137{,}5}{170{,}7825^2}(x-350)\rightarrow y=-0{,}0047x+4{,}4$$

y=0,0047x+4,4 (y sobre x)

Si x=650 mg $\Rightarrow$ y = - 0,0047 · 650+4,4 = 1,35 hrs.

12.

X_i (años)	1	2	3	4	5	6	21
Y_i (piezas)	7	8	6	4	3	2	30
n_i	1	1	1	1	1	1	6
$X_i^2 \cdot n_i$	1	4	9	16	25	36	91
$Y_i^2 \cdot n_i$	49	64	36	16	9	4	178
$X_i \cdot Y_i \cdot n_i$	7	16	18	16	15	12	84

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{21}{6}=3{,}50\ años \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{30}{6}=5\ piezas$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\overline{x}^2}=\sqrt{\frac{91}{6}-3{,}50^2}=1{,}7078\ \textit{años}$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\overline{y}^2}=\sqrt{\frac{178}{6}-5^2}=2{,}1602\ \textit{piezas}$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\overline{x}\cdot\overline{y}=\frac{84}{6}-3{,}50\cdot 5\approx -3{,}5 \qquad r=\frac{\sigma_{xy}}{\sigma_x\sigma_y}\approx -0{,}9487$$

Correlación muy fuerte e inversa

Recta de regresión:

$$y-\overline{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\overline{x})\rightarrow y-5=\frac{-3{,}5}{1{,}7078^2}(x-3{,}50)\rightarrow y=-1{,}2x+9{,}2$$

y= - 1,2x+9,2 (y sobre x)

Si x= 7 años $\Rightarrow$ y = - 1,2 ·7+9,2 = 0,8 piezas.

Si y= 0 piezas $\Rightarrow$ x = 9,2/1,2 = 7,67 años.

También podemos hallar x calculando la recta de regresión de x sobre y.

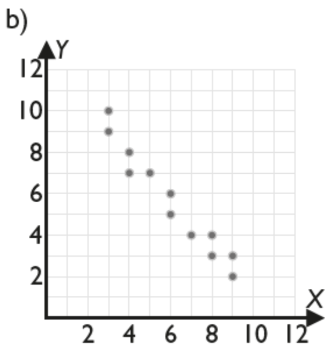
$$x-\overline{x}=\frac{\sigma_{xy}}{\sigma_y^2}\cdot(y-\overline{y})\rightarrow x-3{,}5=\frac{-3{,}5}{4{,}6664}(y-5)\rightarrow x=-0{,}75y+7{,}25$$

x= - 0,75y+7,25 (x sobre y)

Si y= 0 piezas $\Rightarrow$ x = 7,25 años.

La diferencia es muy escasa ya que la correlación es muy fuerte. En caso de que no sea así, es más fiable utilizar la segunda recta de regresión (x sobre y).

15.



Presumiblemente correlación fuerte e inversa. Valor de r próximo a -1. Vamos a calcularlo y comprobaremos la estimación.

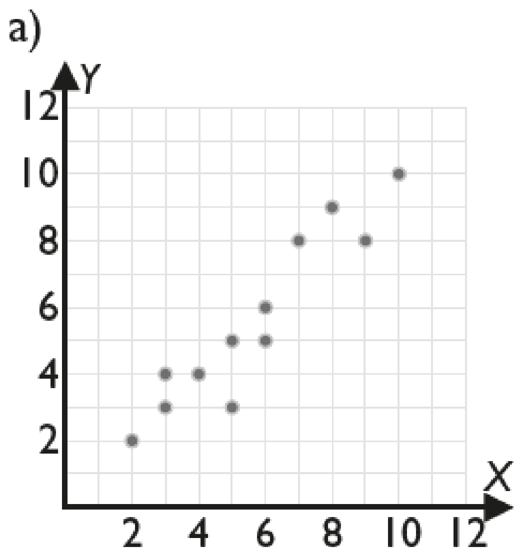
X_i (años)	3	3	4	4	5	6	6	7	8	8	9	9	72
Y_i (piezas)	9	10	7	8	7	5	6	4	3	4	2	3	68
n_i	1	1	1	1	1	1	1	1	1	1	1	1	12
$X_i^2 \cdot n_i$	9	9	16	16	25	36	36	49	64	64	81	81	486
$Y_i^2 \cdot n_i$	81	100	49	64	49	25	36	16	9	16	4	9	458
$X_i \cdot Y_i \cdot n_i$	27	30	28	32	35	30	36	28	24	32	18	27	347

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{72}{12}=6 \text{ años} \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{68}{12}=5{,}6667 \text{ piezas}$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{486}{12}-6^2}=2{,}1213 \text{ años}$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{458}{12}-5{,}6667^2}=2{,}4607 \text{ piezas}$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{347}{12}-6\cdot 5{,}6667 \approx -5{,}0835 \qquad r=\frac{\sigma_{xy}}{\sigma_x \sigma_y}\approx -0{,}9738$$



Presumiblemente correlación fuerte y directa. Valor de r próximo a 1. Vamos a calcularlo y comprobaremos la estimación.

X_i (años)	2	3	3	4	5	5	6	6	7	8	9	10	68
Y_i (piezas)	2	3	4	4	3	5	5	6	8	9	8	10	67
n_i	1	1	1	1	1	1	1	1	1	1	1	1	12
$X_i^2 \cdot n_i$	4	9	9	16	25	25	36	36	49	64	81	100	454
$Y_i^2 \cdot n_i$	4	9	16	16	9	25	25	36	64	81	64	100	449
$X_i \cdot Y_i \cdot n_i$	4	9	12	16	15	25	30	36	56	72	72	100	447

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{68}{12}=5,6667 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{67}{12}=5,5833$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{454}{12}-5,6667^2}=2,3920$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{449}{12}-5,5833^2}=1,8410$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{447}{12}-5,5833\cdot 5,6667\approx 5,6111 \qquad r=\frac{\sigma_{xy}}{\sigma_x \sigma_y}\approx 0,9524$$

17.

X_i	3	3	3	5	5	7	7	7	9	9	9	11	11	
Y_i	24	26	30	26	28	26	28	30	24	28	30	28	30	
n_i	3	2	1	4	3	3	2	1	1	4	3	2	4	33
$X_i \cdot n_i$	9	6	3	20	15	21	14	7	9	36	27	22	44	233
$Y_i \cdot n_i$	72	52	30	104	84	78	56	30	24	112	90	56	120	908
$X_i \cdot Y_i \cdot n_i$	216	156	90	520	420	546	392	210	216	1008	810	616	1320	6250

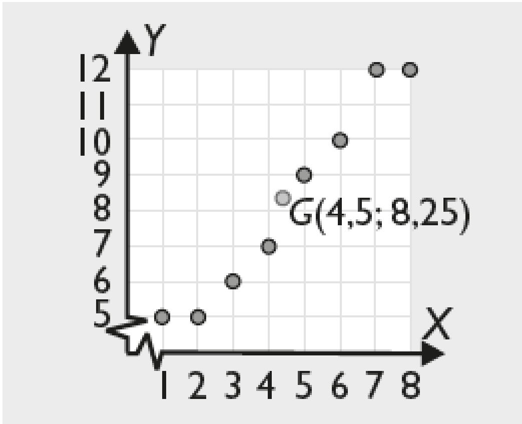
$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{233}{33}=7{,}06 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{908}{33}=27{,}52$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{6520}{33}-7{,}06\cdot 27{,}52\approx 3{,}3475$$

18.

a y b.

X	1	2	3	4	5	6	7	8
Y	5	5	6	7	9	10	12	12



c.

X_i	1	2	3	4	5	6	7	8	
Y_i	5	5	6	7	9	10	12	12	
n_i	1	1	1	1	1	1	1	1	8
$X_i \cdot n_i$	1	2	3	4	5	6	7	8	36
$Y_i \cdot n_i$	5	5	6	7	9	10	12	12	66
$X_i \cdot Y_i \cdot n_i$	5	10	18	28	45	60	84	96	346

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{36}{8}=4{,}5\ grs \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{66}{8}=8{,}25\ cm$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{346}{8}-4{,}5\cdot 8{,}25\approx 6{,}125$$

20.

X_i (ºC)	12	13	13	14	15	16	17	100
Y_i (mm Hg)	800	805	800	803	805	810	810	5633
n_i	1	1	1	1	1	1	1	7
$X_i^2 \cdot n_i$	144	169	169	196	225	256	289	1448
$Y_i^2 \cdot n_i$	640.000	648.025	640.000	644.809	648.025	656.100	656.100	4.533.059
$X_i \cdot Y_i \cdot n_i$	9600	10465	10400	11242	12075	12960	13770	80512

$$\bar{x} = \frac{\Sigma x_i \cdot n_i}{N} = \frac{100}{7} = 14{,}2857\ ^{o}C \qquad \bar{y} = \frac{\Sigma y_i \cdot n_i}{N} = \frac{5633}{7} = 804{,}7143\ mm\ Hg$$

$$\sigma_x = \sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N} - \bar{x}^2} = \sqrt{\frac{1448}{7} - 14{,}2857^2} = 1{,}6661\ ^{o}C$$

$$\sigma_y = \sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N} - \bar{y}^2} = \sqrt{\frac{4.533.059}{7} - 804{,}7143^2} = 3{,}8409\ mm\ Hg$$

$$\sigma_{xy} = \frac{\Sigma x_i \cdot y_i \cdot n_i}{N} - \bar{x} \cdot \bar{y} = \frac{80512}{7} - 14{,}2857 \cdot 804{,}7143 \approx 5{,}8072 \qquad r = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \approx 0{,}9074$$

Correlación muy fuerte y directa

24.

X_i	8,2	8,15	8,4	8,5	8,88	8,81	8,87	8,75	8,87	8,99	86,42
Y_i	4,8	4,83	4,9	4,88	4,95	4,96	4,88	4,8	4,85	4,92	48,77
n_i	1	1	1	1	1	1	1	1	1	1	10
$X_i^2 \cdot n_i$	67,24	66,4225	70,56	72,25	78,8544	77,6161	78,6769	76,5625	78,6769	80,8201	747,6794
$Y_i^2 \cdot n_i$	23,04	23,3289	24,01	23,8144	24,5025	24,6016	23,8144	23,04	23,5225	24,2064	237,8807
$X_i \cdot Y_i \cdot n_i$	39,36	39,3645	41,16	41,48	43,956	43,6976	43,2856	42	43,0195	44,2308	421,554

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{86,42}{10}=8,642 \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{48,77}{10}=4,877$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{747,6794}{10}-8,642^2}=0,2894$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{237,8807}{10}-4,877^2}=0,0542$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{421,554}{10}-8,642\cdot 4,877\approx 0,008366 \qquad r=\frac{\sigma_{xy}}{\sigma_x\sigma_y}\approx 0,54178$$

Recta de regresión:

$$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-4,877=\frac{0,008366}{0,08375}(x-8,642)\rightarrow y=0,0999x+4,0137$$

y=0,0999x+4,0137 (y sobre x)

No sería fiable hacer una estimación ya que la correlación es débil. El valor de r no es próximo, en valor absoluto, a 1.

35.

X_i (cm)	165	168	169	170	170	173	175	175	180	1545
Y_i (cm)	170	170	170	173	173	180	178	172	179	1565
n_i	1	1	1	1	1	1	1	1	1	9
$X_i^2 \cdot n_i$	27225	28224	28561	28900	28900	29929	30625	30625	32400	265389
$Y_i^2 \cdot n_i$	28900	28900	28900	29929	29929	32400	31684	29584	32041	272267
$X_i \cdot Y_i \cdot n_i$	28050	28560	28730	29410	29410	31140	31150	30100	32220	268770

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{1545}{9}=171{,}6667\ cm \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{1565}{9}=173{,}8889\ cm$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{265389}{9}-171{,}6667^2}=4{,}2674\ cm$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{272267}{9}-173{,}8889^2}=3{,}8130$$

$$\sigma_{xy}=\frac{\Sigma x_i \cdot y_i \cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{268770}{9}-171{,}6667\cdot 173{,}8889\approx 12{,}3997$$

a. $$r=\frac{\sigma_{xy}}{\sigma_x \sigma_y}\approx 0{,}76204$$

b. Recta de regresión:

$$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-173{,}8889=\frac{12{,}3997}{18{,}2107}(x-171{,}6667)\rightarrow y=0{,}6809x+57{,}00072$$

y=0,6809x+57,00072 (y sobre x)

c. Si x=185 cm $\Rightarrow$ y = 0,6809 · 185+57,00072 = 182,97 cm.

No sería muy fiable la estimación ya que la correlación es débil. El valor de r no es próximo, en valor absoluto, a 1. $|r| < 0{,}85$.

39.

X_i (horas)	8	7,5	8	8,5	6	7	8	9	62
Y_i (piezas)	3	4	4	5	2	3	5	4	30
n_i	1	1	1	1	1	1	1	1	8
$X_i^2 \cdot n_i$	64	56,25	64	72,25	36	49	64	81	486,5
$Y_i^2 \cdot n_i$	9	16	16	25	4	9	25	16	120
$X_i \cdot Y_i \cdot n_i$	24	30	32	42,5	12	21	40	36	237,5

$$\bar{x} = \frac{\Sigma x_i \cdot n_i}{N} = \frac{62}{8} = 7{,}75\ h \qquad \bar{y} = \frac{\Sigma y_i \cdot n_i}{N} = \frac{30}{8} = 3{,}75\ piezas$$

$$\sigma_x = \sqrt{\frac{\Sigma x^2{}_i \cdot n_i}{N} - \bar{x}^2} = \sqrt{\frac{486{,}5}{9} - 7{,}75^2} = 0{,}866$$

$$\sigma_y = \sqrt{\frac{\Sigma y^2{}_i \cdot n_i}{N} - \bar{y}^2} = \sqrt{\frac{120}{8} - 3{,}75^2} = 0{,}9682$$

$$\sigma_{xy} = \frac{\Sigma x_i \cdot y_i \cdot n_i}{N} - \bar{x} \cdot \bar{y} = \frac{237{,}5}{8} - 7{,}75 \cdot 3{,}75 \approx 0{,}625$$

a. $$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \approx 0{,}7454$$

b. Recta de regresión:

$$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-3{,}75=\frac{0{,}625}{0{,}745}(x-7{,}75)\rightarrow y=0{,}84x-2{,}75$$

y=0,84x-2,75 (y sobre x)

c. Si x=10 h $\Rightarrow$ y = 0,84 $\cdot$ 10-2,75 = 5,65 piezas

No sería muy fiable la estimación ya que la correlación es débil. El valor de r no es próximo, en valor absoluto, a 1. $|r|<0{,}85$.

38.

Estatura	175	180	180	185	183	180	190	175
Peso	77	79	80	82	80	80	85	75

X_i (cm)	175	180	180	185	183	180	190	175	1448
Y_i (kgs)	77	79	80	82	80	80	85	75	638
n_i	1	1	1	1	1	1	1	1	8
$X_i^2 \cdot n_i$	30625	32400	32400	34225	33489	32400	36100	30625	262264
$Y_i^2 \cdot n_i$	5929	6241	6400	6724	6400	6400	7225	5625	50994
$X_i \cdot Y_i \cdot n_i$	13475	14220	14440	15170	14640	14400	16150	13125	115330

$$\bar{x}=\frac{\Sigma x_i \cdot n_i}{N}=\frac{1448}{8}=181\,cm \qquad \bar{y}=\frac{\Sigma y_i \cdot n_i}{N}=\frac{638}{8}=79{,}75\,Kgs$$

$$\sigma_x=\sqrt{\frac{\Sigma x^2{}_i\cdot n_i}{N}-\bar{x}^2}=\sqrt{\frac{262264}{8}-181^2}=4{,}6904$$

$$\sigma_y=\sqrt{\frac{\Sigma y^2{}_i\cdot n_i}{N}-\bar{y}^2}=\sqrt{\frac{50994}{8}-79{,}75^2}=2{,}8174$$

$$\sigma_{xy}=\frac{\Sigma x_i\cdot y_i\cdot n_i}{N}-\bar{x}\cdot\bar{y}=\frac{115330}{8}-181\cdot 79{,}75\approx 12{,}75$$

a. $$r=\frac{\sigma_{xy}}{\sigma_x\sigma_y}\approx 0{,}9648$$

b. Recta de regresión:

$$y-\bar{y}=\frac{\sigma_{xy}}{\sigma_x^2}\cdot(x-\bar{x})\rightarrow y-79{,}75=\frac{12{,}75}{21{,}9999}(x-181)\rightarrow y=0{,}5795x-25{,}1484$$

y=0,5795x-25,1484 (y sobre x)

c. Si x=195 cm $\Rightarrow$ y = 0,5795 · 195-25,1484 = 87,86 Kgs. Como r = 0,96 >0,85 $\Rightarrow$ La correlación es fuerte y la estimación es fiable.